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Trajectory Synthesis for Air Traffic Automation

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Trajectory synthesis algorithms that are key to the center–terminal radar approach control automation system (CTAS) developed at NASA Ames Research Center for air traffic control automation are discussed. CTAS generates computer advisories based on synthesized trajectories that help controllers to produce a safe, efficient, and expeditious flow of traffic over the extended terminal area. Trajectories are synthesized from initial aircraft positions to a metering fix or runway, depending on airspace. The horizontal path is constructed first from specified waypoints using straight lines and constant-radius turns. The vertical trajectory is divided into a series of flight segments. Three types of flight profiles are defined by connecting selected segments in a predetermined order: fast, nominal, and slow. Each profile can produce a certain range of arrival times. A second-order Runge–Kutta scheme is used for integrating a set of simplified point-mass equations to generate vertical trajectories. Then, an iterative scheme is employed to determine the speed that meets a specified arrival time. Several special case trajectories are also explained. Two flight scenarios are used to illustrate the use of trajectory synthesis algorithms.

Nomenclature

C_{D0}, C_{Df}, C_{Dg}, C_{Ds}	= clean, flap, gear, and speed brake drag coefficients, respectively
C_L	= lift coefficient
D	= drag
g	= acceleration of gravity
h	= altitude
L	= lift
M	= Mach number
m	= aircraft mass
R	= turn radius
s	= ground path distance
T	= engine thrust
V_{CAS}	= calibrated airspeed
V_g	= ground speed
V_i	= inertial speed
V_t	= true airspeed
W_l	= horizontal wind magnitude
W_x, W_y, W_h	= wind components in (x, y, h)
w_f	= fuel flow
x, y	= east and north position
α	= angle of attack
α_t	= angle between thrust and relative wind, $\alpha + \varepsilon$
γ_a, γ_i	= air-relative and inertial flight-path angle
δ_F	= flap deflection angle
ε	= engine thrust inclination
θ_c	= crab angle, $\Psi_i - \Psi_a$
θ_{rw}	= relative wind angle, $\Psi_i - \Psi_w$
κ	= engine thrust setting
ϕ_α	= aerodynamic bank angle
Ψ_i, Ψ_a	= (inertial, aerodynamic) heading
Ψ_w	= wind direction

Introduction

FOR the last several decades, the nation's air traffic has increased significantly, and it is projected to grow even further. Major airports in the nation are facing saturation and serious delays. Despite advances in technology, controllers still form advisories for the aircraft based on experience, which requires about three years of training to develop. It is highly desirable to develop computerized automation systems that can reduce controllers' workload and training while guaranteeing flight safety, increasing airport throughput, and reducing average fuel consumption.

There are three types of air traffic control (ATC) authorities: tower; terminal radar approach control (TRACON) or terminal; and air route traffic control center or center. An aircraft must receive the approval from the airport tower before taking off. About 5 miles after leaving the runway, the aircraft enters terminal airspace. About 30 miles beyond the runway, the aircraft enters center airspace. There are 20 centers over the continental United States, and each center has a diameter of about 200 n mile. An aircraft often flies through several centers before reaching its destination. During approach and landing, the aircraft switches from center to terminal and then to tower control at appropriate times, in reverse order of its takeoff. Throughout its flight, an aircraft receives advisories from various air traffic controllers. Controllers' advisories can include speed changes, target altitudes, top of descent locations, and waypoint locations. These advisories guarantee flight safety and sequence aircraft in a certain landing order. At some airports, the controllers are also responsible for having the aircraft cross a specified point at a specified time.

Researchers at NASA Ames Research Center have been developing an ATC automation tool called the Center–TRACON Automation System (CTAS). CTAS computes advisories for aircraft in the center and terminal airspace and displays these to controllers.[1] In essence, CTAS consists of three basic computational functionalities: trajectory synthesizer, scheduler, and conflict resolver. The trajectory synthesizer first computes open-final-time aircraft flight trajectories. Efficient landing times are scheduled by the scheduler, based on estimated times of arrival from these computed trajectories. Then, the trajectories are recomputed to meet the scheduled arrival times. Finally, the trajectories are modified by the conflict resolver to remove any possible conflicts while maintaining the scheduled times of arrival.[2]

CTAS has been demonstrated in practical tests at Dallas–Fort Worth Terminal to be effective in increasing throughput at least 5–10% (Ref. 3). CTAS has also been tested at Denver Center using aircraft equipped with flight management systems, leading to greater

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use of flight management systems and fewer ATC clearances.[4] Currently, the use of datalink to transfer information (e.g., performance data) automatically between air and ground and to transfer more aircraft preferences to the ATC automation is being studied. These plans encompass both center and terminal areas.

This paper explains the principles and details of the current trajectory synthesis algorithms used in CTAS. Essentially, trajectory synthesis predicts aircraft paths using models of aircraft equations of motion so that CTAS can estimate and assign proper landing times and predict and resolve potential conflicts. Therefore, outputs of trajectory synthesis are four-dimensional aircraft trajectories, i.e., three-dimensional aircraft trajectories that meet specified times. To generate these trajectories, aircraft propulsion and aerodynamic performance models, weather, and assumptions of pilot flight techniques are needed. In addition, current aircraft states as initial conditions are required. A desired final time from the scheduler may also be used.

CTAS generates trajectories for all aircraft in a region after every radar sweep, which is approximately every 12 s in center and 4 s in terminal airspace. These trajectories must observe all operational constraints due to assigned routes, aircraft performance limits, ATC procedures, and airline policies, and meet scheduled times of arrival. As a result, trajectory synthesis must solve a highly constrained problem quickly and efficiently.

A procedural approach is modified for generating flight trajectories in CTAS. As its name suggests, this approach produces trajectories that imitate pilot flight procedures. Specifically, a point-mass aircraft model is adopted. The horizontal path is computed first from an approximate vertical path and is approximated by a combination of straight lines and circular arcs. A simplified vertical trajectory is then calculated, divided into a series of flight segments consistent with current-day flight procedures. Some vertical variables are numerically integrated while others are solved algebraically from flight-segment specifications. Iteration between horizontal and vertical trajectories would produce the exact three-dimensional trajectory, but would increase the computation time excessively for ATC automation. Finally, flight speeds are iterated to meet specified arrival times. Trajectories generated by the procedural approach can be specified by controllers via radio and can be flown by pilots of any aircraft. In addition, the procedural approach provides distinct computational advantages over fuel-optimal approaches that could also be used.

The procedural approach of trajectory generation emulates practical pilot procedures. In this approach, the vertical trajectory is divided into a series of flight segments that can be flown by holding constant variables that pilots can read on today's cockpit instruments. Erzberger and Tobias[5] conducted numerical simulations of the procedural approach for time-based air traffic control. Erzberger and Chapel[6] explained the procedural approach in comparison with the fuel-optimal approach. The procedural approach generates speed schedules of constant Mach transitioning to constant calibrated airspeed. This structure is chosen empirically to be consistent with cockpit navigation aids and to be fuel conservative. Izumi[7] examined the effects of different algorithms on airport throughput and fleetwide fuel consumption.

Various authors have studied fuel-optimal approaches for calculating four-dimensional trajectories.[8–12] Williams and Knox[13] compared the two approaches in terms of computational requirements and optimality of trajectories. They showed that the procedural approach produces negligible penalty on optimal fuel cost while using one-third of the computational time as the optimal control approach. The procedural approach is currently used in CTAS for two reasons. It provides a computationally efficient method for generating close-to-optimal trajectories. In addition, the procedural approach accommodates all types of aircraft.

In the rest of the paper, development of algorithms for generating efficient commercial flight paths is reviewed. Equations of motion and methods used in CTAS are presented. Examples of trajectory synthesis are examined.

Equations of Motion

Point-mass equations are adequate for generating aircraft trajectories over a few-minute time span. The three-dimensional point-mass equations of motion for a generic commercial aircraft are

$$\dot V_t = (1/m)(T\cos\alpha_t - D) - g\sin\gamma_a - \dot W_x\cos\gamma_a\sin\Psi_a - \dot W_y\cos\gamma_a\cos\Psi_a - \dot W_h\sin\gamma_a \tag{1}$$

$$V_t\dot\gamma_a = (1/m)(L + T\sin\alpha_t)\cos\phi_a - g\cos\gamma_a + \dot W_x\sin\gamma_a\sin\Psi_a - \dot W_y\sin\gamma_a\cos\Psi_a - \dot W_h\cos\gamma_a \tag{2}$$

$$\dot h = V_t\sin\gamma_a + W_h \tag{3}$$

$$V_g\dot\Psi_i = (L/m)(\sin\phi_a\cos\theta_c + \cos\phi_a\sin\gamma_a\sin\theta_c) + (D/m)\cos\gamma_a\sin\theta_c + (T/m)(\sin\alpha_t\sin\phi_a\cos\theta_c + \sin\alpha_t\cos\phi_a\sin\gamma_a\sin\theta_c - \cos\alpha_t\cos\gamma_a\sin\theta_c) \tag{4}$$

$$\dot x = V_g\sin\Psi_i, \qquad \dot y = V_g\cos\Psi_i \tag{5}$$

$$\dot m = -(\dot w_f/g) \tag{6}$$

where the angles are defined in Fig. 1 and

$$W_l = \sqrt{W_x^2 + W_y^2} \tag{7}$$

$$\tan\Psi_a = \frac{V_g\sin\Psi_i - W_x}{V_g\cos\Psi_i - W_y} \tag{8}$$

$$V_g = V_i\cos\gamma_i = \sqrt{(V_t\cos\gamma_a)^2 - (W_l\sin\theta_{rw})^2} - W_l\cos\theta_{rw} \tag{9}$$

$$\sin\theta_c = \frac{W_l\sin\theta_{rw}}{V_t\cos\gamma_a} \tag{10}$$

$$V_i = \sqrt{V_g^2 + (V_t\sin\gamma_a + W_h)^2} \tag{11}$$

$$\tan\gamma_i = \frac{V_t\sin\gamma_a + W_h}{V_g} \tag{12}$$

Several approximations are made for ATC automation. The angle of attack and flight-path angle are commonly assumed small. The crab angle is assumed to be small with respect to θ_{rw}. This assumption breaks down in high wind situations. Several angular rates are also considered to be small. The rate of change of the flight-path angle is set to zero, assuming quasisteady flight. The angular rate of the velocity track angle Ψ_i is assumed to be small. The time when Ψ_i is changing is short because most of the flight is composed of straight segments, connected by brief turns. During turns the approximations are less accurate.

Because of the lack of sufficient weather information, the vertical component of wind is assumed to be zero. The weather information is currently updated every 3 h with plans to increase the update rate to once per hour. Because this is about the same amount of time an aircraft spends under CTAS control, the wind is assumed to be stationary in time. In addition, the wind at the aircraft is interpolated from a set of values at different horizontal locations,[14] but the partial derivatives with respect to x and y are assumed to be zero.

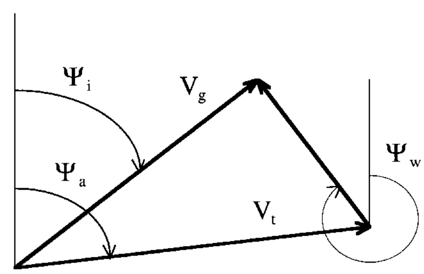


Fig. 1 Definition of angles.

Finally, the mass of the aircraft is assumed to be constant throughout the flight. A standard descent weight for each type of aircraft is used.

The combination of these assumptions removes the effect of two of the independent variables, m and γ_a, reducing Eqs. (1–6) to

$$\dot{V}_t = \frac{T - D}{m} - g\gamma_a - \frac{\mathrm{d}(W_l \cos\theta_{rw})}{\mathrm{d}t} \tag{13}$$

$$L = mg \tag{14}$$

$$\dot{h} = V_t\gamma_a = V_g\gamma_i \tag{15}$$

$$\dot{\Psi}_i = \frac{L\sin\phi_a}{mV_g} \tag{16}$$

$$\dot{x} = V_g \sin\Psi_i, \qquad \dot{y} = V_g\cos\Psi_i \tag{17}$$

$$\dot{m} = 0 \tag{18}$$

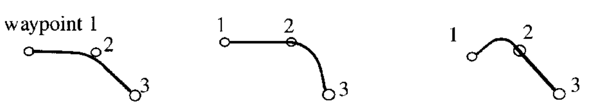
Horizontal Trajectory

The current ATC system prevents conflicts among all aircraft by assigning each aircraft a specified horizontal route composed of a series of points. However, the horizontal path and vertical path are coupled through ground speed. In particular, the radius of each turn is $R = V_g^2/g\sin\phi_a$. For a given Mach or V_{CAS}, the ground speed is a function of altitude and wind. For simplicity, a constant ground speed is assumed during a turn. A constant bank angle of 20 deg for center and 30 deg for terminal is used. Then, the horizontal path can be constructed from straight lines and circular arcs, based on an approximate vertical profile. The horizontal and vertical paths are separated to simplify the equations and lead to faster solutions. With a sensible approximation to the vertical profile, the full three-dimensional profile will be acceptably close to the separate horizontal and vertical profiles.

The approximate vertical profile assumes a 1000-ft descent for every 3-n mile horizontal path distance. The path is generated from the endpoint backward to the initial altitude to determine an approximate top of descent. The airspeed is assumed to be the cruise V_{CAS} before the top of descent and the descent V_{CAS} in descent, ignoring accelerations and decelerations. With an approximate altitude and V_{CAS} profile, a standard-atmosphere approximation for temperature and pressure and zero wind, the ground speed at any path distance can be calculated. The ground speed for a turn is found by taking Ψ_i at the middle of the turn and an average of the altitudes and airspeeds from the two endpoints and middle of the turn.

Once the turn radii are known and turn arcs are connected to straight line segments geometrically, the angle Ψ_i is defined, and so Eq. (16) need not be integrated. This is the horizontal profile used for synthesizing the vertical profile. Furthermore, Eq. (17) is replaced by a single equation for the path distance $s = (\dot{x}^2 + \dot{y}^2)^{1/2}$. The positions x and y are calculated from the integrated s and the known Ψ_i. The horizontal path could be refined using a more accurate vertical trajectory. Such an iteration doubles the computational time and does not significantly change the horizontal path.

There are three currently standard types of turns at a waypoint defined by CTAS: inside turns, start-at turns, and end-at turns (Fig. 2). The most commonly used turn is the inside turn (Fig. 2a). Many aircraft travel along routes that are defined as bearings from a point. When an aircraft changes from one route to another, the pilot simply changes his heading to intercept the new bearing. An inside turn is constructed by placing a circular arc with the calculated radius tangent to the lines connecting the three waypoints. Start-at turns are used when the aircraft has an initial turn or cannot turn until it is past some restricted airspace. Figure 2b shows an initial turn where waypoint two is the location of the aircraft and waypoint one is on an imaginary line backward along the aircraft's track angle. Start-at turns are constructed from a circular arc, which starts tangent to the line connecting waypoints 1 and 2 and ends when the track angle is pointing at the next waypoint. End-at turns are used if the aircraft must pass through a specific point with a particular heading. For example, an aircraft must be heading in the direction of the runway to land. Figure 2c shows a final turn where waypoint two is the beginning of the runway and waypoint three is the end. End-at turns are constructed with the same method as start-at turns, in the opposite direction.



a) Inside turn **b) Start-at waypoint 2** **c) End-at waypoint 2**

Fig. 2 Turn types.

These turn types were designed to emulate pilots changing between desired headings. Most turns calculated by flight management systems can be reproduced in CTAS by creating combinations of these turns. One combination currently available in CTAS is designed to avoid disregarding too much of the route. The initial point of an inside turn is limited to be at most twice the turn radius from the waypoint, which causes the turn to overshoot the next route. The turn starts a distance of two turn radii from the waypoint, and the aircraft intersects the next route with a 30-deg angle.

Flight Segments

The vertical trajectory is composed of a series of flight segments to emulate manual pilot procedures and create a simple set of advisories to issue by radio. The segments are designed to hold constant variables for which the pilot has instruments. The speed variables are issued as advisories, whereas the other variables are procedural. For jet aircraft, an idle-thrust procedure is widely accepted and implemented in flight management systems. For turboprops and piston aircraft, the accurate use of CTAS requires defining a standard pilot procedure.

Segment Definition

There are three types of variables that can be held constant by a pilot: engine control (idle thrust, maximum thrust), speed (Mach or V_{CAS}), and vertical rate ($\dot{h}$ or γ_i). A vertical flight segment is defined by holding two of these variable types constant. The values of the constant variables must be within the performance limits of the aircraft.

There are three possible combinations of the two variables. These combinations hold constant engine control and speed, vertical rate and engine control, and vertical rate and speed. Furthermore, segments are divided into separate categories by the choice of the speed variable (Mach or V_{CAS}). Finally, accelerations or decelerations are divided into separate categories by whether or not they occur in level flight. As a result, there are five types of segments: cruise segment, level acceleration or deceleration segment, Mach descent or ascent segment, V_{CAS} descent or ascent segment, and descending or ascending acceleration segment. These segments are summarized in Table 1.

Exactly which variables are held constant in a segment depends on the type of aircraft, the type of segment, and the standard pilot procedure. A *crossover altitude* is defined for every aircraft where the true airspeed calculated from the maximum descent V_{CAS} equals the true airspeed calculated from the maximum descent Mach. Speed changes are issued by the controller as Mach above the crossover altitude and V_{CAS} below. A cruise segment has $\dot{h}$ or equivalently γ_i equal to zero. The level acceleration or deceleration segment holds thrust at idle for deceleration and at maximum cruise for acceleration

Table 1 Segment definitions

Segment	Integration	Constants
Cruise	s	Vertical rate and speed
Level acceleration/ deceleration	s, V_t	Engine control and vertical rate
Mach descent/ascent	s, h	Mach and engine control or γ_i
V_{CAS} descent/ascent	s, h	V_{CAS} and engine control or γ_i
Descending or ascending acceleration	s, h, V_t	Engine control and vertical rate

Table 2 Capture definitions

Segment	Integration direction	Capture
Cruise	Forward/backward	s
Level acceleration/ deceleration	Forward/backward	V_{CAS} or Mach
Mach descent	Forward	V_{CAS} or h
Mach descent	Backward	h
Mach ascent	Forward	h
V_{CAS} descent	Forward	h
V_{CAS} descent	Backward	Mach or h
V_{CAS} ascent	Forward	Mach or h
Descending acceleration	Forward	Mach or V_{CAS}
Ascending acceleration	Forward	V_{CAS}

and zero vertical rate. In a Mach descent segment, jet aircraft use idle thrust descents, whereas prop aircraft use a constant γ_i of 2.35 deg. This procedure for props is based on a 1000-ft descent in 4 n mile and was defined by examining live radar track data of turboprop aircraft and choosing an average descent rate. This is shallower than the jet rule of 1000 ft in 3 n mile, because turboprops do not descend with idle thrust. In a Mach-ascent segment, jets use maximum cruise thrust and props use maximum climb thrust. The V_{CAS} descent/ascent segment has the same conditions as the Mach descent/ascent segment except that V_{CAS} is held constant instead of Mach. The descending acceleration segment holds idle thrust, and a constant altitude rate of -3000 ft/min for jets and constant inertial flight-path angle of 2.35 for props. The ascending acceleration segment holds constant altitude rate, dependent on aircraft type, and constant maximum cruise or climb thrust, depending on aircraft type.

For each segment the variables that must be integrated are shown in Table 1. The remaining variables can be calculated explicitly from the constant variables. For example, when Mach or V_{CAS} is held constant, the true airspeed can be solved algebraically, and Eq. (13) does not need to be integrated. Similarly, Eq. (15) is not integrated when altitude is held constant.

Segment Connection

The end of a segment is defined by a capture condition. A capture condition occurs when a specific nonconstant variable reaches some desired value.[15] For example, the cruise segment ends at a desired position. The precision that defines a successful capture is achieving an accuracy within 0.0003 for Mach, 0.1 kn for V_{CAS}, 20 ft for altitude, and 0.005 n mile for path distance. These values are designed for the precision of the computer variables and are more precise than can be measured by current instruments. Table 2 lists all capture variables for each type of segment.

The cruise segment always ends at a path distance. The level deceleration or acceleration segment ends at either a desired Mach or V_{CAS}, depending on whether the altitude is above or below the crossover altitude. The Mach descent segment ends at the descent V_{CAS} in forward integration, and at the top-of-descent altitude in backward integration. The Mach ascent segment ends at top of ascent altitude.

The V_{CAS} descent segment captures altitude when integrated forward. When the segment is integrated backward, it captures Mach if the cruise altitude is above the crossover altitude and captures top-of-descent altitude otherwise. If the top of ascent altitude is below the crossover altitude calculated using ascent speeds, the V_{CAS} ascent segment captures altitude; otherwise it captures Mach.

The descending acceleration segment is only integrated forward and may capture either Mach or V_{CAS}, depending on whether the cruise altitude is above the crossover altitude. The ascending acceleration segment only occurs at low altitudes and always captures V_{CAS}.

Trajectory Construction

A complete vertical trajectory is created by combining segments in a specified order using templates. The templates contain three types of descent and cruise profiles: fast, nominal, and slow. A fast cruise profile has a cruise speed higher than the initial speed, requiring an initial acceleration. A fast descent profile has a descent speed higher than the cruise speed, requiring a descending acceleration. A slow cruise profile has a cruise speed slower than the initial speed, requiring an initial deceleration. A slow descent profile has a descent speed slower than the cruise speed, requiring a deceleration before the top of descent. The nominal cruise profile consists of cruise speed equal to the initial speed, whereas the nominal descent profile has neither acceleration nor deceleration segments. These types of descent and cruise profiles are defined to produce a time range for the trajectory. Then the speeds are iterated to meet a specified final time. In contrast, there is only a single defined nominal ascent profile, because ascent profiles are more constrained by aircraft performance. According to Federal Aviation Administration (FAA) regulations, aircraft below 10,000 ft should not exceed a speed of 250-kn indicated airspeed. Therefore, any trajectory that begins or ends below 10,000 ft accelerates or decelerates, if necessary, to 250-kn V_{CAS} or lower at 10,000 ft.

Descent Profiles

The descent portion of the trajectories is integrated backward from the final values of position, altitude, and speed, except for the descending–acceleration segment, which is integrated forward. A general fast descent template consists of a descending acceleration to the descent Mach, followed by a constant Mach segment to the descent V_{CAS}, followed by a constant V_{CAS} segment to the final altitude, followed by a level deceleration to the final speed. The fastest trajectory uses the maximum descent Mach and V_{CAS}. The junction between the acceleration segment and the constant Mach segment is not known a priori and is determined using a three-step approximation. First, the constant Mach segment is integrated backward to the initial altitude producing an approximate top of descent. Next, a descending acceleration segment is integrated forward from the approximate top of descent to the descent Mach. Finally, the path distance at the end of the acceleration segment is shifted to match the path distance of the constant Mach segment at the same altitude. If the aircraft is below the crossover altitude, the constant Mach segment is removed, and the acceleration segment ends at the descent V_{CAS}.

Nominal and slow trajectories are entirely integrated backward. The nominal template is the same as the fast, except the acceleration segment is removed. Nominal descent profiles are bound by the fast nominal and the slow nominal. In the fast-nominal profile, the aircraft descends at the cruise Mach. The descent V_{CAS} is found from the cruise Mach using a linear relationship, defined for each aircraft type by taking the highest descent Mach and V_{CAS} and a typical descent Mach and V_{CAS} as two points forming a line. This relationship couples descent Mach and V_{CAS} into a single variable for iteration to meet desired final time. The slow-nominal descent profile descends at the cruise V_{CAS}. Below the crossover altitude, fast nominal and slow nominal reduce to a single nominal descent.

A general slow descent template consists of a level deceleration to the descent V_{CAS}, followed by a constant V_{CAS} segment to the final altitude, followed by a level deceleration to the final speed. The slow template is the same for all aircraft at all altitudes. The slowest trajectory uses the minimum descent V_{CAS}.

Cruise Profiles

CTAS can produce cruise speed advisories to help meet a specified time or can respond to a desired cruise speed entered by the controller. CTAS can also respond to altitude changes entered by the controller. When either a cruise speed or altitude change is present, the cruise section of the trajectory is integrated forward from the initial position to the top of descent.

When there is no altitude change, and the cruise speed is faster than the initial speed (from radar), the cruise profile is calculated from three segments. First, there is a level flight segment at the initial speed with a 30-s duration to model controller and pilot response time. Then there is an acceleration or deceleration segment to the cruise speed, followed by level flight at the cruise speed to the top of descent. The fastest cruise profile uses the maximum cruise speed, whereas the slowest cruise profile uses the slowest cruise speed; both of which are defined for each aircraft type.

Profiles with Changes in Cruise Altitude

When there is an change to a lower altitude, but no cruise speed change, the descent is either at constant Mach or constant V_{CAS}, depending on whether the new altitude is above or below the crossover altitude. The descent is a single segment with constant speed and constant inertial flight-path angle. The flight-path angle for immediate altitude changes can be set as an input to the algorithm. At the end of the descent, there is a level flight segment at the cruise speed to the top of descent. The standard pilot descent assumption of 1000 ft for every 3 n mile is the current default value.

When there are both a lower altitude and a cruise speed change, there are three possibilities. If the cruise V_{CAS} is lower than the initial V_{CAS}, there is a level flight deceleration to the cruise airspeed followed by a constant-flight-path angle/constant-airspeed descent to the new cruise altitude. If the cruise Mach is higher than the initial Mach, there is a constant-Mach/constant-flight-path angle descent to the new cruise altitude, followed by a level flight acceleration to the cruise Mach. If the cruise V_{CAS} is higher than the initial V_{CAS} but the cruise Mach is lower, the trajectory follows a constant-Mach, constant-flight-path angle segment at the initial Mach until the cruise V_{CAS} is captured, followed by a constant-V_{CAS} constant-flight-path angle segment to the initial altitude. All of these descents are followed by a level flight segment at the cruise speed to the top of descent.

If there is a change to a higher altitude and the initial altitude is less than 10,000 ft, the first segment is an acceleration to the 250-kn speed limit at a specified vertical speed. The vertical speed is set to 1500 ft/min for jet aircraft, 1000 ft/min for turboprops, and 750 ft/min for piston aircraft. The next segment is a constant-V_{CAS} ascent to 10,000 ft, followed by a level flight acceleration to the climb V_{CAS}. The engine control is always set to the maximum cruise condition for jet aircraft or the maximum climb value for turboprops and pistons.

The next segment in the ascent profile is a constant-V_{CAS} ascent segment, which ends either at the cruise altitude or at the ascent Mach limit. The ascent Mach limit is set to 95% of the cruise Mach. If the ascent V_{CAS} at the cruise altitude is less than the ascent Mach limit, the constant V_{CAS} ascent segment continues to the cruise altitude. If the cruise V_{CAS} is higher than the climb V_{CAS}, there is a level flight acceleration at the top of ascent, followed by a level flight segment to the top of descent or the end of the route if there is no descent.

If the ascent speed at the cruise altitude is higher than the ascent Mach limit, the constant-V_{CAS} ascent segment ends when the ascent Mach limit is reached. Next, there is a constant-Mach ascent segment to the cruise altitude, followed by a level flight acceleration to the cruise speed, followed by a level flight segment to the top of descent or end of trajectory.

Integration

Vertical trajectory synthesis in CTAS currently uses a second-order Runge–Kutta method[16] for a balance between accuracy and computational speed. The integration step size is chosen from the type of segment being integrated and whether the aircraft is in a turn. The default step sizes are 60 s for a constant V_{CAS} or Mach descent or ascent segment, 30 s for a deceleration or acceleration, and 300 s for a cruise segment. If the aircraft is in a turn, the step size is chosen so that either the turn will be completed in one step or the aircraft will turn 30 deg in one step, whichever is smaller. If the aircraft is approaching a turn, the step size is chosen so that the beginning of the turn will be reached in one step.

Each segment is integrated until the capture condition is passed. Then the step size is decreased by linearly interpolating between the last two steps. The trajectory is reintegrated from the previous step with the new step size. This cycle continues until either a maximum number of iterations is reached or the capture condition is met. The logic assumes that the variable being captured changes monotonically from its initial value to the capture value. If the capture variable changes direction during the integration, it is checked for an extremum in the step just integrated. To find whether an extremum exists, the equations are integrated with a step size of 1/10 the previous step size. If the value of the capture variable approaches the capture value, the extremum is assumed to be in this segment. The step size is reduced to 90% of its original value, and the previous step is integrated again. The step size reduction continues as long as the capture variable diverges, until a maximum number of iterations is reached. If the capture variable changes direction with a step size of 1/10, trajectory synthesis produces an error.

Input Data Sources

Four types of inputs are required for trajectory synthesis. They are the aircraft state, trajectory constraints, aircraft performance models, and atmospheric conditions.

Initial aircraft position is defined from radar measurements of aircraft position and altitudes from aircraft transponders. The FAA computer estimates ground speed, track angle, and vertical rate and sends them to CTAS every 12 s in the center and every 4 s in the terminal. Ground speed, track angle, and vertical rate are found by taking differences of the measurements and are, therefore, noisy and inaccurate. Ground speed is used to find the initial V_{CAS} of the aircraft, which is maintained during the entire cruise for aircraft if no other cruise speed is specified. Inaccurate ground speeds can cause a large error in these trajectories. Track angle errors produce the initial-path errors. Altitude rate is not used in trajectory synthesis, and so its accuracy is unimportant. Flight plans define the type of aircraft and the planned future state, including route. They are used upon receipt from the airlines.

Constraints on trajectories are desired final time and horizontal route as a series of points, which are produced by the CTAS program, and vertical constraints (i.e., desired altitudes and speeds), which come from ATC constraints, flight plans, and controller preferences. CTAS maintains procedure data files for airlines and aircraft type. Each airline has a set of desired descent speeds. Performance constraints are defined for each aircraft type.

The aircraft performance data is split into three files: aircraft specific, aerodynamic drag, and engine. The aircraft-specific file consists of a set of speeds and weights for every aircraft identified by the FAA. For example, it contains the maximum and minimum cruise and descent speeds of the aircraft that define the fastest and slowest profiles and the speeds that define flap settings and gear deployment. The default descent speed contained in this file is used if there is no airline procedure descent speed. The slope and intercept of the descent Mach-V_{CAS} line are also in the aircraft-specific file. Each aircraft type is cross referenced to an aerodynamic drag model and an engine model. There are currently 425 aircraft types recognized by the FAA.

The aerodynamic drag file contains three- and four-dimensional tables of the values of the drag coefficient. The drag coefficient is defined for the clean configuration, various flap settings, with the gear down, and with speed brakes if available. Mathematically,

$$C_D = C_{D0}(M, C_L) + C_{Df}(\delta_F, C_L) + C_{Dg}(\delta_F, M, C_L) + C_{Ds}(M)$$

CTAS now has aerodynamic drag models for 15 aircraft.

The engine file has tables of thrust for each engine. For every engine, the idle thrust is given as a function of altitude and airspeed. Values of the maximum cruise thrust and maximum climb thrust are also given as a function of altitude and temperature. Currently, CTAS has propulsion models for 10 different engines.

The atmospheric predictions consist of horizontal wind, pressure, temperature, and geometric altitude at a horizontal grid throughout the center. At each gridpoint the weather data are given at various pressure altitudes. The information is currently updated every 3 h, but there are plans to produce an hourly forecast between the weather updates. Weather values at each integration step are found by interpolating linearly between the nearest eight points forming a cube.[14] Currently, experiments are underway to update the grid with aircraft wind measurements.

Meeting a Desired Time

CTAS is a time-based ground system for ATC automation. Synthesized trajectories are required to meet scheduled times of arrival, produced by the CTAS scheduler, at a boundary point between the center and terminal, called the metering fix, and at the runway. By scheduling aircraft, CTAS promises to reduce flight at lower altitudes and thus save fleetwide fuel consumption.

To meet a desired arrival time, the fastest, slowest, and nominal trajectories are generated. If the desired time is shorter than the fastest time, the fastest profile is used. If it is longer than the slowest time, the slowest profile is used. Otherwise, the desired time is between either the fastest and nominal trajectories or the slowest and nominal trajectories. The interpolation is, therefore, performed only between trajectories that have the same segment types.

There are several speed modes that can be chosen to meet the time. These vary only descent speed, only cruise speed, cruise speed then descent speed, or cruise and descent speed together. The descent-only mode varies descent speed to meet the time with cruise speed held constant at either the initial speed or an input cruise speed. The cruise-only mode varies cruise speed with descent speed held constant at either the default speed or an input speed. The cruise-then-descent mode varies cruise speed first; then if that is not enough to meet the time, varies descent speed. The cruise-and-descent mode tries to keep the two speeds as close as possible. If the default values of the two are different, they are first matched, then varied together until an extremum value of one speed is reached; then the other is changed to its extremum if necessary. In contrast, flight management systems pick the speeds from a table that optimizes a specified combination of fuel and time, known as the direct operating cost.

The speed values that meet the final time are found by iteration. Because speeds are rounded to 5 kn, the final time only needs to be met within 20 s. Up to four iterations are allowed before a diagnostic message is produced suggesting an infinite loop.

Special Cases

Three special trajectory-synthesis modes where the previous logic is simplified and two procedures to handle situations where the logic breaks down are now discussed.

Simplified Logic Modes

A special mode, called the specified-top-of-descent mode, was designed to compare trajectories calculated in CTAS to trajectories calculated by a flight management system. A major difference between the two is the location of the top of descent, because this is a function of aircraft and atmospheric model assumptions. In this mode, the trajectory is integrated forward from a top of descent, input by a user, through the final segment. The end of the trajectory may occur after the end of the route. If it occurs before the end of the route a level flight segment is added. The descent may be made at either idle thrust or fixed flight-path angle.

The overflight mode exists to create trajectories for aircraft that are cruising through the airspace at constant altitude. This mode calculates a level flight segment following the horizontal route.

In the terminal, because of the low altitudes and slow speeds, the aircraft dynamics are not as important. The trajectories calculated all use kinematic constant-γ_i descents. There is also a further simplification of a constant deceleration rate used. For further details, see Ref. 17.

Failed Logic Procedures

If the acceleration segment in a fast profile ends below the final altitude, the small-altitude-difference procedure is used. This only occurs for aircraft where the initial altitude is very close to the final altitude. The small-altitude-difference profiles are integrated backward. If the initial altitude is below the crossover altitude, the template consists of cruise at current speed, followed by a level acceleration to the descent V_{CAS}, followed by a constant V_{CAS} descent to the final altitude, followed by a level deceleration to the final speed. If the altitude is above the crossover altitude, the template consists of a cruise at current speed to the top of descent, followed by a constant Mach descent at cruise Mach to the final-path distance. This trajectory crosses the final point at the desired altitude but not the desired speed.

The top of descent is normally calculated in a low-drag configuration. If the calculated top of descent is farther back than the initial position, the descent is recalculated with speed brakes (if available). When the aircraft cannot descend to the desired altitude even with speed brakes, the relaxed-metering-fix procedure is used. This can occur when the aircraft is too high to meet the crossing condition, or if the modeled drag is too small. The procedure tries to meet the final speed but not the final altitude. First, an approximate deceleration distance is calculated from the descent V_{CAS} using a rule of 1 n mile to decelerate 10 kn. If this distance is greater than the initial distance, the trajectory decelerates immediately until the final-path distance or final speed is reached. Otherwise, the relaxed-metering-fix procedure integrates forward from the initial condition following the fast, nominal, or slow template until the deceleration-path distance is reached, then decelerates until the final distance or speed is reached. If the final speed is reached first, there is a level cruise segment to the final-path distance. It is also possible for the small-altitude-difference procedure to produce a top of descent farther back than the initial condition. For this case the small-altitude-difference template is integrated forward using the relaxed-metering-fix procedure.

Examples of Trajectory Synthesis

The first example describes the fastest, slowest, and nominal trajectories for a Boeing 737-200 aircraft varying both cruise and descent speed. The aircraft starts with a V_{CAS} of 275 kn at an altitude of 31,000 ft and a course of 90 deg. The final conditions at the metering fix are an altitude of 11,000 ft and a V_{CAS} of 210 kn. There is no wind.

The horizontal profile for the slow trajectory is shown in Fig. 3. Four waypoints are specified making two 90-deg turns. They have coordinates (0, 0), the initial condition, (50, 0), (50, 50), and (100, 50), the metering fix. The input points are shown as circles. The synthesized horizontal trajectory is shown as a solid line. The turns are circular arcs between the turn endpoints, which are shown as an $\times$. The radius of the first turn is 5.9 n mile based on the slowest cruise V_{CAS} of 240 kn. The top of descent occurs at the square between the two turns, so the radius of the second turn is based on the slowest descent speed of 220 kn and is 4.5 n mile. The total path distance traveled by the aircraft is 145.5 n mile. For the nominal trajectory, the radii of the turns are 7.6 and 6.7 n mile, due to the faster cruise and descent speeds, both equal to 275 kn, and the total path distance is 143.8 n mile. For the fastest trajectory, the radii are 8.0 and 8.8 n mile, due to a cruise speed of 281 kn and a descent speed of 340 kn, and the path distance is 142.8 n mile.

The vertical trajectories for the four cases are described in Table 3. Altitude is plotted vs time in Fig. 4. The fastest descent is the steepest

Table 3 Summary of trajectory results

Trajectory type	Cruise V_{CAS}, kn	Top of descent, n mile	Descent Mach	Descent V_{CAS}, kn	Time, s
Slowest	240	70.0	N/A	220	1537
Nominal	275	65.3	N/A	275	1290
Fastest	281	52.1	0.8	340	1188

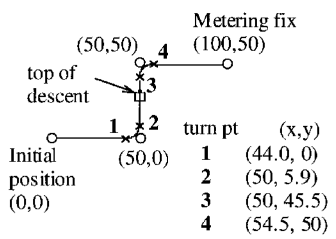


Fig. 3 Horizontal path input to trajectory synthesis: ○, waypoint and ×, turn endpoint.

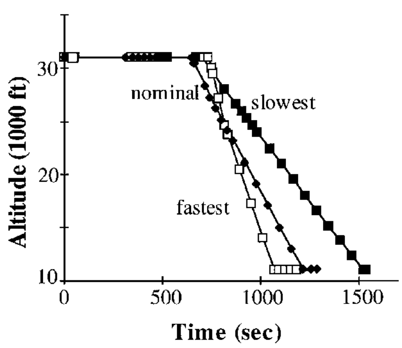


Fig. 4 Altitude vs time.

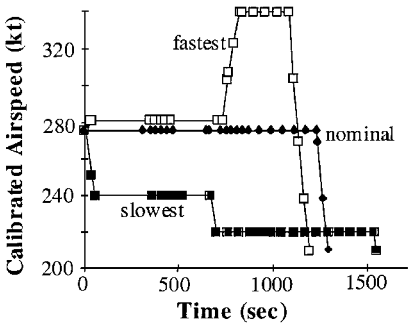


Fig. 5 Calibrated airspeed vs time for fastest, nominal, and slowest examples.

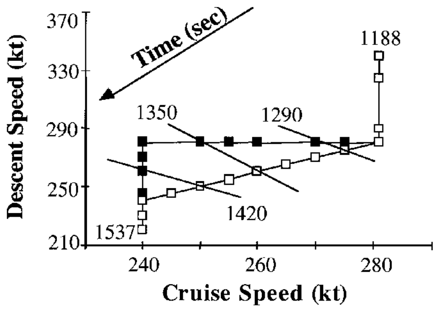


Fig. 6 Meet-time descent speed vs cruise speed.

due to the larger descent speed, causing the top of descent to be closest to the final point. The slowest descent is the shallowest with the nominal trajectory in the middle. The squares represent integration steps, which occur at segment boundaries and at integration steps within the segments. The set of points between 400 and 500 s is the first turn. A plot of V_{CAS} vs time is shown in Fig. 5. At the end of all trajectories, the aircraft descends to 11,000 ft and slows to 210 kn.

The second example shows meet-time speed profiles for the same aircraft using the two modes that vary both cruise and descent V_{CAS} (Fig. 6). The times to be met vary between 1188 and 1537 s. The two modes are cruise then descent, which is shown in the solid boxes, and cruise and descent. The cruise-then-descent mode varies cruise speed with a constant descent V_{CAS} of 280 kn (the company profile speed) until the cruise speed is saturated. Then descent speed is varied to meet time. The cruise-and-descent mode varies the two speeds together until the cruise speed is saturated, then varies descent speed. Looking at a final time of 1350 s, the cruise-then-descent mode has a cruise V_{CAS} of 250 kn followed by a descent V_{CAS} of 280 kn. The cruise-and-descent mode has a cruise and descent speed equal to 260 kn. The advantage of the cruise-and-descent mode is that the aircraft does not need to slow down then speed up.

However, the cruise-and-descent mode cannot meet the desired time of arrival as accurately in the linear region because the two speeds must be equal and are rounded to 5 kn.

Conclusion

The principles and details of the trajectory synthesis algorithms used in CTAS are explained. Trajectories are generated for one aircraft at a time from the current position to a metering fix or runway. These trajectories are used to form advisories that can help controllers produce a safe and efficient flow of traffic. Trajectories are consistent with flight procedures and operational requirements. The trajectories are computed by a fast-time integration of a set of simplified point-mass equations of motion. The trajectory generation uses inputs of aircraft states, real-time weather, models of aircraft drag and propulsion performance, specified waypoints for horizontal routes, company preferred operation procedures, and ATC and aircraft constraints.

Because of the simplifications, the trajectory synthesis algorithms are performed very quickly, so that trajectories for many aircraft can be calculated. On a Sun Sparcstation 20, a trajectory with no altitude changes takes 15 ms of processing time. A trajectory with a descent or ascent takes 30 ms, and a full ascent, cruise, and descent trajectory takes 120 ms. The iteration necessary to meet time takes an average of 200 ms. Work is underway to optimize the code, which is expected to make these calculations at least twice as fast.

The trajectories have been validated in field tests to be very close to trajectories flown by aircraft given commands and calculated by flight management systems. Using the trajectory calculations, with the CTAS scheduler and conflict resolver, produces at least 5–10% improvement in aircraft throughput. The system also allows greater use of flight management systems.

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